Engineering Mathematics and Physics Department Mathematics 1 Code: Math 101 Time Allowed: 2 hours	Modern University For Technology & Information	Academic year: 2010/2011 Semester: Autumn Final Exam: 19 / 1 / 2011 Examiner: Dr. Mohamed Eid
Answer 4 questions only	**Faculty of Engineering**	**Marks**

Question 1

Find y\` from the following: **10**

(a) $y = 2x^4 + \ln(2 + \sinh x)$ (b) $y = 3^x.\sin x$ (c) $y = [2x + \cos x]^8$

(d) $y = 2^{\sin x} + \cos x$ (e) $y^3 + x + \sin(xy) = 0$

Question 2

(a)Find the limits: (i) $\lim\limits_{x \to 0} \dfrac{2x}{\tan 3x}$ (ii) $\lim\limits_{x \to \infty} \dfrac{x + 2^x}{x^2 + 3^x}$ (iii) $\lim\limits_{x \to 0} \dfrac{3^x - 1}{\ln(1 + x)}$ **6**

(b)Compute the length of the curve $y = 1 + \frac{2}{3} x^{3/2}$, x in [0, 1]. **4**

Question 3

(a)Determine the maximum and minimum values of $f(x) = 2x^3 - 24x + 1$ **4**

(b)Find the inflection points of $f(x) = 2 + (x-1)^3$ **3**

(c)Compute the integral $\int \dfrac{x^3}{\sqrt{4 - x^2}} dx$ **3**

Question 4

Find the following integrals: **10**

(a) $\int (x^2 + 2^x)\, dx$ (b) $\int \dfrac{x + \cos x}{x^2 + 2\sin x} dx$ (c) $\int \dfrac{2x}{(x^2+4)^6} dx$

(d) $\int x \cos x\, dx$ (e) $\int_0^1 \dfrac{2x + 1}{x^2 + 4x + 3} dx$

Question 5

(a)Find the area of the region between the curve $y = x^2 - 1$, x– axis, x in [0, 2] **3**

(b)If the region between the curve $y = \sin x$, x-axis, $x \in [0, \pi/2]$ is rotated about x-axis. Find the volume of the generated solid. **3**

(c) Find the integral $\int (\sin^2 x . \cos^3 x)\, dx$ **4**

Good Luck *Dr. Mohamed Eid*

Model Answer

[1] (a) $y` = 8x^3 + \frac{0+\cosh x}{2+\sinh x}$

(b) $y` = 3^x \cos x + 3^x \ln 3.\sin x$

(c) $y` = 8[2x + \cos x]^7.(2 - \sin x)$

(d) $y` = 2^{\sin x} \ln 2.\cos x - \sin x$

(e) Differentiate with respect to x, we get $3y^2 y` + 1 + \cos(xy).(xy` + y) = 0$

Then $y`[3y^2 + x\cos(xy)] = -1 - y\cos(xy)$. Hence $y` = \frac{-1 - y\cos(xy)}{3y^2 + x\cos(xy)}$

[2](a)(i) $\lim_{x\to 0} \frac{2x}{\tan 3x} = \frac{2}{3}$

(ii) $\lim_{x\to\infty} \frac{x+2^x}{x^2+3^x} = \lim_{x\to\infty} \frac{\frac{x}{3^x} + (\frac{2}{3})^x}{\frac{x^2}{3^x} + 1} = \frac{0+0}{0+1} = 0$

(iii) $\lim_{x\to 0} \frac{3^x - 1}{\ln(1+x)} = \lim_{x\to 0} \frac{3^x - 1}{x} \frac{x}{\ln(1+x)} = \ln 3.1 = \ln 3$

(b) Since $y` = \sqrt{x}$. Then $L = \int_0^1 \sqrt{1 + (y`)^2} dx = \int_0^1 \sqrt{1+x}\, dx = \frac{2}{3}(1+x)^{3/2} = \frac{2}{3}(2^{3/2} - 1)$

[3] (a) Since $f`(x) = 6x^2 - 24$. Then $6x^2 - 24 = 0$ Or $x^2 - 4 = 0$.
Then $x = \pm 2$ are critical points. Since f ``(x) = 12x.
Since f ``(2) = 24, then the point 2 is minimum
Since f ``(– 2) = – 24, then the point – 2 is maximum

(b)Since $f`(x) = 3(x-1)^2$, f ``(x) = 6(x – 1). Then x = 1.
Since f ``($1^-$) = f ``(0) = – 6 and f ``($1^+$) = f ``(2) = 6.
Hence the point x = 1 is inflection.

(c)Put x = 2sin t, dx = 2 cos t dt. Then

$$\int \frac{x^3}{\sqrt{4-x^2}} dx = 8\int \sin^3 t dt = 8\int \sin t(1 - \cos^2 t) dt = 8[-\cos t + \frac{1}{3}\cos^3 t] + c$$

$$= 8[-\cos(\sin^{-1} x/2) + \frac{1}{3}\cos^3(\sin^{-1} x/2)] + c$$

[4](a) $\int (x^2 + 2^x)\, dx = \frac{x^3}{3} + \frac{2^x}{\ln 2} + c$

(b) $\int \frac{x + \cos x}{x^2 + 2\sin x}\, dx = \frac{1}{2}\ln(x^2 + 2\sin x) + c$

(c) $\int \frac{2x}{(x^2+4)^6}\, dx = \int 2x(x^2+4)^{-6} dx = \frac{1}{-5}(x^2+4)^{-5} + c$

(d) Using parts method: $\int x\cos x\, dx = x\sin x + \cos x + c$

(e) Using partial fraction method:

$$\int_0^1 \frac{2x+1}{x^2+4x+3} dx = \int_0^1 \left(\frac{5/2}{x+3} - \frac{1/2}{x+1}\right) dx = \frac{5}{2}\ln(x+3) - \frac{1}{2}\ln(x+1)$$

$$= \frac{5}{2}(\ln 4 - \ln 3) - \frac{1}{2}(\ln 2 - \ln 1)$$

[5](a)Since $x^2 - 1 = 0$ when $x = 1$ lies in [0, 2]

Then $A = \int_0^1 (x^2 - 1) dx + \int_1^2 (x^2 - 1) dx = -\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$

(b) Volume $= \int_0^{\pi/2} \pi [f(x)]^2 dx = \pi \int_0^{\pi/2} \sin^2 x\, dx = \frac{\pi}{2} \int_0^{\pi/2} (1 - \cos 2x)\, dx$

$$= \frac{\pi}{2}\left(x - \frac{\sin 2x}{2}\right) = \frac{\pi}{2}\left(\frac{\pi}{2} - 0\right) = \frac{\pi^2}{4}$$

(c) $I = \int \sin^2 x . \cos^2 x . \cos x\, dx = \int \cos x . \sin^2 x . (1 - \sin^2 x) dx = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + c$

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Question 1

Find y` from the following: — 10

(a) $y = 3x^2 + \ln(x + \sin x)$ (b) $y = e^x.\cosh x$ (c) $y = [x + \tan x]^5$

(d) $y = 3^x + \log x$ (e) $2x + y^2 + 2^y + 3^x = 0$

Question 2

(a)Find the limits: (i) $\lim\limits_{x \to 0} \dfrac{\sin 2x}{\tan 3x}$ (ii) $\lim\limits_{x \to 1} \dfrac{\ln x}{x - 1}$ (iii) $\lim\limits_{x \to \infty} \dfrac{x^2 - 1}{x^3 + x}$ — 6

(b)Find the area of the region between the curve $y = x^3$, x– axis, x in [– 1, 1] — 4

Question 3

(a)Determine the maximum and minimum values of $f(x) = x^3 - 12x + 2$ — 4

(b)Find the inflection points of $f(x) = 1 + (x-2)^4$ — 3

(c)Compute the integral $\int \dfrac{x^2}{\sqrt{9 - x^2}} dx$ — 3

Question 4

Find the following integrals: — 10

(a) $\int (3x^2 - 3^x)\, dx$ (b) $\int \dfrac{\cos x - \sin x}{\cos x + \sin x} dx$ (c) $\int x.3^x\, dx$

(d) $\int \dfrac{x^2}{(x^3+1)^5} dx$ (e) $\int_1^2 \dfrac{2x + 1}{x^2 + 5x + 6} dx$

Question 5

(a)Compute the length of the curve $y = x^{3/2}$, x in [1, 2] — 3

(b)If the region between the curve $y = x^2 + 3$, x-axis, x in [0, 1] is rotated about x-axis. Find the volume of the generated solid. — 3

(c) Find the integral $\int (\sin^3 x.\cos^4 x)\, dx$ — 4

Good Luck *Dr. Mohamed Eid*

Name: .

(1)Determine the even and odd functions:

(a) $f(x) = 3x^2 + 4$ (b) $f(x) = 2x^3 + 4x$ (c) $f(x) = 2^x + 3x$

(1) Find the following limits:(a) $\lim_{x \to 1} \frac{x^3 - 1}{x - 2}$ (b) $\lim_{x \to 2} \frac{\sqrt[3]{x} - \sqrt[3]{2}}{x^2 - 4}$ (c) $\lim_{x \to 0} \frac{2^x - 1}{3x}$

(2)Find y` where: (a) $y = 4x^3 + \log x$ (b) $y = 8^x \sin x$ (c) $y = \frac{\cos x}{x^2 + 3}$

Answer

Mid – Term Exam Time 80 Minutes Mathematics 1 10 – 11 – 2010

(1)Determine the domain of the functions:

(a) $f(x) = x^3 - \frac{4}{x^2 - 9}$ (b) $g(x) = 3^x + \frac{4}{x^2 + 3}$

(2) Find the limits: (a) $\lim_{x \to 2} \frac{x^2 - 4}{x - 3}$ (b) $\lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{\sqrt[4]{x} - 1}$ (c) $\lim_{x \to 0} \frac{\tan x}{x^2 + 2x}$

(3) Find y\` where: (a) $y = x^3 . \sin x$ (b) $y = 8^x + \tan x$

(c) $y = \log x + \ln x + \ln(x + \cos x)$ (d) $y = \cos 3x + \cosh x^3$

(4) Find y\` from the equation $x^2 + y^3 + y \sin x = 0$

(5) Find the maximum and minimum values of the functions:

(a) $f(x) = x^3 - 12x + 1$ (b) $f(x) = \ln x$

(6) Find the inflection points of the function $f(x) = x^3 - 12x^2 + 1$

Good luck ***Dr. Mohamed Eid***

Name: Quiz III

(1)Find the following integrals:

(i) $\int \sqrt{9-x^2}\, dx$ (ii) $\int \frac{x^3}{\sqrt{4-x^2}}\, dx$ (iii) $\int_0^2 (x+2^x)dx$

(iv) $\int_{-1}^{1} \frac{x^3}{\sqrt{2-x^2}}dx$ (v) $\int_0^{\pi/2} \frac{1}{4+5\sin x}\, dx$

Answer